# An Overview on the Contributions of the Academician Octav Onicescu to the Informational Statistics and Further Developments

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Abstract

*Octav Onicescu is the greatest statistician that Romania ever had. He is the only Romanian statistician included in the Statisticians of the Centuries' volume published in 2001 by Springer Science at the initiative of the International Statistical Institute. The paper presents an overview on the academician Octav Onicescu's research activity in the field of informational statistics. Two major contributions brought to this domain are discussed: Onicescu's informational energy and correlation, highlighting some applications' areas and further developments.*

**Keywords**: Probability theory, Information theory, Mathematical statistics, Informational statistics, Onicescu's Informational energy

## 1 Introduction

Octav Onicescu (1892-1983) is one of the greatest mathematicians of Romania with a remarkable international reputation, being the only Romanian statistician included in the Statisticians of the Centuries' volume published in 2001 by Springer Science at the initiative of the International Statistical Institute, the representative association for international statistics, his presentation in the book being made by Marius Iosifescu [11]. He considered himself ([18], page 18) as *a researcher of facts: human, social, economic, of natural phenomena, with mathematical means, preferable probabilistic or mechanical, researcher that determined along his whole life to assimilate much more mathematical science in order to use it in his research activity.* The academician Octav Onicescu promoted probability as the science of events measure and random processes, and mechanics as support or model of any science of natural movements. He is the founder of the probability theory and statistics in Romania and he developed a new mechanics, the *invariantive mechanics* [20]. Also, he investigated and applied methods from geometry, pure algebra, functional analysis, game theory, mathematical logic and topology. Details regarding the Octav Onicescu's education, academic and research activities are given in [3], [11], [17] and [18].

In 1930 (the year of the population census in Romania), Octav Onicescu founded together with other scientists (e.g. Gheorghe Mihoc, Alex Pantazi, Nicolae Teodorescu, Miron Nicolescu, Grigore Moisil, Alex Froda, Nicolae Ciorănescu, Max Sanielevici, Paul Sterian, Roman Moldovan, I. Argeșeanu, N. Georgescu-Roegen, S. Manoilă, Mircea Vulcănescu, Ciril Petrescu), the *School of Statistics, Actuarial Science and Computation* that became the *Institute of Statistics* till 1948, when it was dismantled, and it was later reactivated as a department of the Institute of

Mathematics of the Romanian Academy [3], [11]. This institute represents one of the greatest scientific achievements of his generation that was dedicated to the society service in an efficient and direct way.

Onicescu looked for new significances of the statistical information. His main contributions to the science are in the domains of probability theory, statistics, mechanics, some domains of analysis, philosophy etc. He had many collaborators and was the mentor of several remarkable mathematicians (*his first doctoral student* Gheorghe Mihoc, who was the closest and probable, the dearest disciple and collaborator, Silviu Guiașu, Cassius Ionescu Tulcea, Marius Iosifescu, Ion Văduva, Vasile Ștefănescu, Mihai Botez, Ion Săcuiu, Tiberiu Postelnicu, Luminița State, just naming a few of them).

At University of Trieste he taught several lessons about the *information theory*, trying to adopt as information measurement instrument, *the informational energy* instead of the Shannon's entropy. Still, in order to demonstrate the existence of codes (founded by the Shannon's theorem) he had to use the entropy, which although more complicated as expression, gave a simpler demonstration. He emphasized the quality of informational energy as a statistical instrument.

The results of the academician Octav Onicescu' research work were published in several research articles, studies, memoires, textbooks and books (more than 200). In this article we have included only 15 references, from which one refers to his memoirs [17], one to his life's pathways [18] and the rest to his scientific activity in the domains of probability theory, statistical information, invariantive mechanics etc.

The academician Octav Onicescu had a complex personality, that apart his academic, scientific and cultural activities (described in [3], [17], [18]), was an excellent organizer (being the president of some professional societies, scientific seminars and the manager of certain public institutions in Romania and abroad) that participated actively during the first World War to the Mărășești battle in 1917 (exactly one century ago), being the leader of a company within the Romanian air force. He was a great professor with remarkable pedagogical and methodical abilities that was admired by his students. Octav Onicescu is an excellent model as a scientist, as a professor and as a human being, for every mathematician, every professor, every researcher and even everyone, which contributed to the development of science at national and international level.

From the vast research activity of academician Onicescu, in this paper we focus on some achievements in the domain of informational statistics and their applications.

## 2 A brief overview on the Onicescu's informational statistics theory

Informational statistics is an area of mathematical statistics that study the structure of the statistical relations and their correlations. Shannon [34] proposed the entropy $(-\sum_{i=1}^{n} p_i \log p_i)$ as a measure of the uniformity degree or diversity degree of the corresponding distribution. The entropy has the additive property for independent structures. Onicescu demonstrated that from the statistical viewpoint, the more simple expression $\sum_{i=1}^{n} p_i^2$, which he named as *informational energy* characterizes as well as the Shannon's entropy, the uniformity or diversity of a distribution. Also, he proved the possible extensions to infinite distributions of this characteristic and revealed the statistical significances of the information correlation. The concept of *informational energy* was introduced by academician Onicescu in 1966 ([23], [24]) being the basis of the theory he built, namely *the Onicescu's theory* on informational statistics.

We present some elements of the Onicescu's theory of informational energy and correlation ([19], [23] and [35]).

**Definition 1.** (*informational energy*) Suppose $x$ is a random variable with the distribution of probabilities: ($p_1$, $p_2$, ..., $p_n$). The *informational energy* of $x$, denoted as $IE(x)$, is defined according to [23] as follows:

(1) $$IE(x) = \sum_{i=1}^{n} p_i^2$$

This expression was introduced first by Corrado Gini and it was named by Octav Onicescu as the *informational energy*. Relation (1) is named the *Onicescu formula* for the measurement of uncertainty or information related to $x$. It can be written as relation (2).

(2) $$IE(p_1, p_2, \ldots, p_n) = \sum_{i=1}^{n} p_i^2 .$$

The *informational energy* is a measure of the uncertainty of an events system, $S$, with $n$ events (considered as an *experiment*) having the corresponding frequencies $f_1$, $f_2$, ..., $f_n$, and is denoted as follows:

$$IE(S) = \sum_{i=1}^{n} f_i^2; \quad \sum_{i=1}^{n} f_i = 1.$$

Some properties of the informational energy are:

1) If the events are equiprobable then $IE(S) = \dfrac{1}{n}$.
2) If one event has the probability 1 and the others have the zero probability, then $IE(S) = 1$.
3) $IE(S) \in \left[\dfrac{1}{n}, 1\right]$.
4) If we add to $S$ an event with zero probability, the informational energy is not modified.

Another important concept of the Onicescu's theory is *the conditioned informational energy*.

**Definition 2.** (*conditioned informational energy*) Suppose we have two experiments $A$ and $B$, where $B$ is conditioned by $A$. The probability distribution, $g_{ij}$, is defined by the following relation:

(3) $$g_{ij} = p(B_i / A_j), \; i = 1,2,\ldots,n; \, j = 1,2,\ldots,m$$

The definition of *the conditioned informational energy* of the $B$ experiment when within the $A$ experiment occurred the $A_j$ event, is given by the following expression:

$$IE(B/A_j) = \sum_{i=1}^{n} g_{ij}^2$$

The core concept of the informational statistics is the *informational correlation*, which was introduced by Octav Onicescu, and is a coefficient that has a remarkable property, that *its equality to 1 represents the distributions identity*.

**Definition 3.** (*informational correlation*) Let's consider two experiments $A$ and $B$, characterized by an $n$-system with $n$ events $A_1, A_2, ..., A_n$ and $B_1, B_2, ..., B_n$, with the following probability distributions:

$$p(A_1) = p_1,\ p(A_2) = p_2,\ \ldots,\ p(A_n) = p_n;$$
$$p(B_1) = q_1,\ p(B_2) = q_2,\ \ldots,\ p(B_n) = q_n.$$

The correlation between $A$ and $B$, denoted as $IC(A,B)$, is given by relation (4).

(4) $$IC(A,B) = \sum_{i=1}^{n} p_i q_i$$

Some properties of the informational correlation are given as follows.

1) $0 \le IC(A,B) \le 1$;
2) $IC(A,A) = \sum_{i=1}^{n} p_i^2 = IE(A)$, i.e. the informational correlation of an experiment with itself is the informational energy.
3) $IC(A,B) \le IE(A) \bullet IE(B)$

The *informational correlation* is a measure of the connection between two systems of events having $n$ common characteristics.

**Definition 4.** (*indifferent experiments*) Two experiments $A$ and $B$ with the probability distribution ($p_1, p_2, ..., p_n$) and ($q_1, q_2, ..., q_n$) are *indifferent* if there informational correlation is zero, i.e. $IC(p_1, p_2, ..., p_n, q_1, q_2, ..., q_n) = 0$.

Let's consider the following four experiments $A, B, C$ and $D$, where $A$ is connected to $C$ and $B$ is connected to $D$. Then, $IC(A\times C, B\times D) = IC(A,B) \bullet IC(C,D)$, where $\times$ is the product operator of two experiments and is defined later.

The experiments $A$, $B$, $C$ and $D$ are given by the following corresponding probability distributions ($p_1, p_2, ..., p_n$), ($q_1, q_2, ..., q_n$), ($r_1, r_2, ..., r_m$) and ($s_1, s_2, ..., s_m$).

**Definition 5.** (*the product of two experiments*) $A \times C$ is the experiment $A$ and $C$ product that has the probability distribution given by relation (5):

(5) $$A\times C = \begin{pmatrix} A_1C_1 & \cdots & A_1C_m & \cdots & A_nC_1 & \cdots & A_nC_m \\ p_1r_1 & \cdots & p_1r_m & \cdots & p_nr_1 & \cdots & p_nr_m \end{pmatrix}$$

where, $A_i$ ($i=1, ..., n$), $C_j$ ($j=1, ..., m$) are the events of the experiments $A$ and $C$, and $p_i$, $r_j$ are the probabilities corresponding to these events.

Let's consider two experiments $A$ and $B$ characterized by the probabilities ($p_1, p_2, ..., p_n$) and ($q_1, q_2, ..., q_n$). Suppose that the $C$ experiment is conditioned by the results of the $A$ experiment and the $D$ experiment is conditioned by the results of the $B$ experiment.

If under the $A$ experiment it was produced the $A_i$ event then the events of the $C$ experiment will obviously depend on the probability of $A_i$ occurring under the $A$ experiment and we'll have:

$$C/A_i = \begin{pmatrix} C_1/A_i & C_2/A_i & \cdots & C_m/A_i \\ r_{i1} & r_{i2} & \cdots & r_{im} \end{pmatrix}$$

where $P(C_1/A_i)=r_{i1}$, $P(C_2/A_i)=r_{i2}$, ..., $P(C_m/A_i)=r_{im}$. The same for $B$ and $D$.

**Definition 6.** (*conditioned informational correlation*) The conditioned informational correlation of the experiments $C$ and $D$ when the events $A_i$ and $B_j$ occurred is given by relation (6).

$$IC(C/A_i, D/B_j) = IC(r_{i1},...,r_{im}; s_{j1},...,s_{jm}) = \sum_{k=1}^{m} r_{ik} s_{jk} \quad (6)$$

**Definition 7.** (*correlation coefficient*) The correlation coefficient of two experiments $A$ and $B$, denoted as $R(A,B)$, is given by the following expression:

$$R(A,B) = \frac{\sum_{i=1}^{n} p_i q_i}{\sqrt{\left(\sum_{i=1}^{n} p_i^2\right)\left(\sum_{i=1}^{n} q_i^2\right)}} = \frac{IC(A,B)}{\sqrt{IE(A) \bullet IE(B)}} \quad (7)$$

Examples of properties of the correlation coefficient:

1) $R(A,B) = R(B,A)$,

2) $R(A,B) \in [0, 1]$.

Examples of other concepts that were introduced by the Onicescu's theory are: *multiple correlation* and *multiple correlation coefficient*. More details on Onicescu's theory are included in [22], [26], [29] and some of the references discussed in the next section.

The research work of academician Onicescu on informational statistics was continued by his collaborators and disciples. We have selected the contributions of three mathematicians, Gheorghe Mihoc, Ion Văduva and Luminița State, presented in [35].

1. Gheorghe Mihoc proposed some estimators of the informational energy and formulated some theorems (*Gheorghe Mihoc' theorems*).
2. The statistical hypotheses verification with information energy was studied by Ion Văduva who formulated his own theory (*Ion Văduva' theory*).
3. The behavior of the Onicescu' informational energy corresponding to a posteriori distribution and the study on the Onicescu' informational energy corresponding to the weighting processes were the main subjects of the research performed by Luminița State in the area of informational statistics who formulated some theorems (*Luminița State' theorems*).

In the next section we shall present some details regarding these contributions and more applications will be discussed.

## 3 Applications of the Onicescu's informational statistic theory

The new systematic theory on informational statistics developed by academician Octav Onicescu starting from the informational energy concept introduced by him in 1966 was applied to several domains during the last fifty years. We have selected for this review some further developments and applications.

*Further developments (selection)*

1. Professor Gheorghe Mihoc noticed the connection between *IE* and the dispersion index of the probability values distribution $p_i$ ($i=1, ..., n$) of an experiment. If in relation (1) the probabilities $p_i$ are replaced with frequencies $f_i$ ($i=1, ..., n$) then it is obtained the *empirical information energy*. This new concept introduced by Gheorghe Mihoc opened new perspectives of the Onicescu's theory.

**Definition 8.** (*empirical information energy*) The empirical information energy of an experiment defined by the frequencies $f_i$ is given by the expression:

$$IE(f_1, f_2,..., f_n) = \sum_{i=1}^{n} f_i^2 \quad (8)$$

Two theorems that were given by Gheorghe Mihoc are included in [35].

The mathematician Gheorghe Mihoc brought several contributions to the informational statistics and probability theory (see. e.g. [15], [27], [28], [30]), some of them in collaboration with academician Octav Onicescu and other mathematicians (as e.g. C. Ionescu Tulcea).

2. Another important contribution to further developments of the Onicescu’s theory on informational statistics was the introduction of the *weighted entropy* and *weighted energy* concepts by Silviu Guiaşu.

**Definition 9.** (*weighted informational energy*) The weighted informational energy of an experiment $A$ having the elementary events $A_1, A_2, ..., A_n$ ($p(A_i)=p_i$; $\Sigma p_i = 1$) to whom correspond the weights $w_1, ..., w_n$ is given by the expression:

$$IE(w_1, w_2,...,w_n; p_1, p_2,..., p_n) = \sum_{k=1}^{n} w_k \cdot p_k^2 \quad (9)$$

$$IE(w_1, w_2,...,w_n; p_1, p_2,..., p_n) \geq= 0$$

One further development of this contribution was performed by Luminița State, who proposed two theorems (included in [35]).

Another contribution of Silviu Guiaşu is the definition of the informational energy for continuous random variables. Also, together with the academician Octav Onicescu he built *the first theory on random automata* [8], [25] and published a book on statistical mechanics [21]. Also, Silviu Guiaşu developed an information model of learning theory [10] and presented some applications of information theory in [9].

3. We have selected five other mathematicians that collaborated with academician Octav Onicescu in the area of informational statistics and further developed and applied their research work, Ion Văduva, Marius Iosifescu, Mihai Botez, Vasile Ştefănescu and Luminița State. Ion Văduva is the first researcher who used *IE* for *checking statistical hypotheses*. He formulated a theory on this subject. Also, he gave examples of industrial applications of this theory for technological processes. Some of his contributions are described in [36] (a book on dispersional analysis) and [37]. Marius Iosifescu extended some results previously obtained by Onicescu and Mihoc to the case of chains with complete connections whose variables can take a finite number of values, showing that the reduced sum of variables checks at limit the normal law. Also, he extended the concept of a Markov chain’ entropy to chains with complete connections, his research on sampling entropy for random homogeneous systems with complete connections being published in [12]. Mihai Botez worked with the academician Octav Onicescu to the development of *information econometrics* and they published a book on this subject, for uncertainty and economic modelling [16]. Vasile Ştefănescu collaborated with the academician Onicescu in the area of informational statistics, focusing on applications, some results being published in [19], [35], and he continued the research work by introducing new concepts as *the measure of information loss* concept. Luminița State introduced the definition of a *σ-experiment* and used the *a posteriori distribution series* providing some theorems (as e.g. those given in [35]).

*Applications (selection)*

4. *Applications of the informational correlation to the study of the education efficiency* (see e.g. [35]). In such applications, the informational energy was used for the detection of the factors that can influence the educational processes (as e.g. the student's level of knowledge, the student's learning style, the professor's competences, the teaching style) in order to build some effective pedagogical and methodical strategies that improve the outcomes of the educational activity. Another application of the Onicescu's theory in the educational domain is the use of the informational energy of a questionnaire (as defined by Claude Picard) in order to evaluate students' knowledge and to improve the quality of the educational processes.

5. *Applications of the informational energy to qualitative factorial analysis, to the agriculture domain* (e.g. planning the territorial distribution of the agriculture production), *in linguistics* (e.g. the phonetic structure of a language, for automated translations between natural languages). Some examples are given in [19], [35].

6. *Applications in social sciences* (e.g. the distribution of the population by age in different countries) and *economy*. Examples of some recent published research work in these areas are the use of informational energy on measuring external complexity of complex adaptive systems [5], its use in financial time series [6], the approach proposed in [33] for financial econometrics, and the uniformity test based on informational energy described in [32].

7. *Applications in biology, physics, chemistry and engineering*. An analysis on the use of the Onicescu's informational energy in some fundamental physical models is presented in [1]. Other examples of using Onicescu's informational energy in recent research work performed in physics and physical chemistry are discussed in [2] and [38], while some applications in engineering are described in [6] (for sensor data analytics), in [14] (as a measure of energy consumption), in [4] (first use in an artificial neural network model for adaptive resonance theory, the new fuzzy ARTMAP architecture) and in [7] (tackling the inference of the informational energy from small datasets). The last two applications are based on the $o(X,Y)$ estimator (given by relation (10), introduced by the authors in some previous work).

$$o(X,Y) = IE(X/Y) - IE(X) \quad (10)$$

Finally, at the end of this brief overview, we mention that some *applications of informational statistics in medicine and petrochemical industry* are presented in the PhD theses elaborated by two mathematicians from Petroleum-Gas University of Ploiești, Miron Oprea and Cristian Marinoiu, under the supervision of two disciples of the academician Octav Onicescu, Tiberiu Postelnicu and Ion Văduva, respectively, at the Institute of Mathematics of Romanian Academy and the University of Bucharest. The PhD thesis of Miron Oprea [31] tackles some stochastic aspects in the diagnosis processes focusing on medical diagnosis. The PhD thesis of Cristian Marinoiu [13] presents models of linear regression and their applications, mainly to the catalytic cracking process.

**Conclusion**

The academician Octav Onicescu is the greatest Romanian statistician with a remarkable international recognition. One of his main achievement in the domain of informational statistics is *the Onicescu's theory of informational energy and correlation*. His research work was continued by its collaborators and disciples, followed by several PhD researchers that elaborated PhD theses with topics in this area, providing new applications in various fields such as engineering, education, medicine, sociology, economy, for different types of processes and systems. The Onicescu's theory can still inspire nowadays researchers in new current research directions as we have seen in this brief overview that included recent published work or by searching the ScienceDirect web site with his research work citations.

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